

Table 17.6

ELLIPTIC INTEGRAL OF THE SECOND KIND $E(\varphi|\alpha)$

$$E(\varphi|\alpha) = \int_0^\varphi (1 - \sin^2 \alpha \sin^2 \theta)^{\frac{1}{2}} d\theta$$

$\alpha \backslash \varphi$	0°	5°	10°	15°	20°	25°	30°
0	0	0.08726 646	0.17453 293	0.26179 939	0.34906 585	0.43633 231	0.52359 878
2	0	0.08726 633	0.17453 185	0.26179 579	0.34905 742	0.43631 608	0.52357 119
4	0	0.08726 592	0.17452 864	0.26178 503	0.34903 218	0.43626 745	0.52348 856
6	0	0.08726 525	0.17452 330	0.26176 715	0.34899 025	0.43618 665	0.52335 123
8	0	0.08726 432	0.17451 587	0.26174 224	0.34893 181	0.43607 403	0.52315 981
10	0	0.08726 313	0.17450 636	0.26171 041	0.34885 714	0.43593 011	0.52291 511
12	0	0.08726 168	0.17449 485	0.26167 182	0.34876 657	0.43575 552	0.52261 821
14	0	0.08725 999	0.17448 137	0.26162 664	0.34866 055	0.43555 106	0.52227 039
16	0	0.08725 806	0.17446 599	0.26157 510	0.34853 954	0.43531 765	0.52187 317
18	0	0.08725 590	0.17444 879	0.26151 743	0.34840 412	0.43505 633	0.52142 828
20	0	0.08725 352	0.17442 985	0.26145 391	0.34825 492	0.43476 831	0.52093 770
22	0	0.08725 094	0.17440 926	0.26138 485	0.34809 262	0.43445 488	0.52040 357
24	0	0.08724 816	0.17438 712	0.26131 056	0.34791 800	0.43411 749	0.51982 827
26	0	0.08724 521	0.17436 353	0.26123 141	0.34773 187	0.43375 767	0.51921 436
28	0	0.08724 208	0.17433 862	0.26114 778	0.34753 510	0.43337 709	0.51856 461
30	0	0.08723 881	0.17431 250	0.26106 005	0.34732 863	0.43297 749	0.51788 193
32	0	0.08723 540	0.17428 529	0.26096 867	0.34711 342	0.43256 075	0.51716 944
34	0	0.08723 187	0.17425 714	0.26087 405	0.34689 050	0.43212 880	0.51643 040
36	0	0.08722 824	0.17422 817	0.26077 666	0.34666 093	0.43168 368	0.51566 820
38	0	0.08722 453	0.17419 852	0.26067 697	0.34642 580	0.43122 748	0.51488 638
40	0	0.08722 075	0.17416 835	0.26057 545	0.34618 625	0.43076 236	0.51408 862
42	0	0.08721 692	0.17413 779	0.26047 261	0.34594 343	0.43029 055	0.51327 866
44	0	0.08721 307	0.17410 700	0.26036 893	0.34569 850	0.42981 431	0.51246 037
46	0	0.08720 920	0.17407 613	0.26026 492	0.34545 266	0.42933 594	0.51163 767
48	0	0.08720 535	0.17404 531	0.26016 110	0.34520 710	0.42885 776	0.51081 454
50	0	0.08720 152	0.17401 472	0.26005 795	0.34496 302	0.42838 212	0.50999 501
52	0	0.08719 774	0.17398 449	0.25995 600	0.34472 162	0.42791 134	0.50918 310
54	0	0.08719 402	0.17395 477	0.25985 574	0.34448 409	0.42744 775	0.50838 287
56	0	0.08719 039	0.17392 571	0.25975 765	0.34425 159	0.42699 368	0.50759 831
58	0	0.08718 686	0.17389 745	0.25966 224	0.34402 529	0.42655 138	0.50683 341
60	0	0.08718 345	0.17387 013	0.25956 996	0.34380 631	0.42612 308	0.50609 207
62	0	0.08718 017	0.17384 388	0.25948 126	0.34359 575	0.42571 097	0.50537 811
64	0	0.08717 704	0.17381 883	0.25939 660	0.34339 465	0.42531 712	0.50469 523
66	0	0.08717 408	0.17379 511	0.25931 640	0.34320 404	0.42494 358	0.50404 700
68	0	0.08717 130	0.17377 283	0.25924 104	0.34302 487	0.42459 224	0.50343 686
70	0	0.08716 871	0.17375 210	0.25917 090	0.34285 805	0.42426 495	0.50286 804
72	0	0.08716 633	0.17373 302	0.25910 634	0.34270 443	0.42396 339	0.50234 359
74	0	0.08716 416	0.17371 568	0.25904 767	0.34256 478	0.42368 913	0.50186 633
76	0	0.08716 223	0.17370 018	0.25899 519	0.34243 984	0.42344 363	0.50143 886
78	0	0.08716 053	0.17368 659	0.25894 917	0.34233 022	0.42322 817	0.50106 351
80	0	0.08715 909	0.17367 498	0.25890 983	0.34223 650	0.42304 389	0.50074 232
82	0	0.08715 789	0.17366 539	0.25887 737	0.34215 915	0.42289 175	0.50047 707
84	0	0.08715 695	0.17365 789	0.25885 195	0.34209 857	0.42277 258	0.50026 923
86	0	0.08715 628	0.17365 250	0.25883 370	0.34205 507	0.42268 700	0.50011 993
88	0	0.08715 588	0.17364 926	0.25882 271	0.34202 889	0.42263 547	0.50003 003
90	0	0.08715 574	0.17364 818	0.25881 905	0.34202 014	0.42261 826	0.50000 000
		$\begin{bmatrix} (-8)4 \\ 3 \end{bmatrix}$	$\begin{bmatrix} (-7)3 \\ 4 \end{bmatrix}$	$\begin{bmatrix} (-7)9 \\ 4 \end{bmatrix}$	$\begin{bmatrix} (-6)2 \\ 5 \end{bmatrix}$	$\begin{bmatrix} (-6)4 \\ 5 \end{bmatrix}$	$\begin{bmatrix} (-6)7 \\ 5 \end{bmatrix}$
5	0	0.08726 562	0.17452 624	0.26177 698	0.34901 329	0.43623 105	0.52342 670
15	0	0.08725 905	0.17447 391	0.26160 165	0.34860 188	0.43543 791	0.52207 785
25	0	0.08724 671	0.17437 550	0.26127 157	0.34782 632	0.43394 028	0.51952 597
35	0	0.08723 006	0.17424 275	0.26082 567	0.34677 648	0.43190 776	0.51605 197
45	0	0.08721 113	0.17409 157	0.26031 693	0.34557 562	0.42957 525	0.51204 932
55	0	0.08719 220	0.17394 015	0.25980 639	0.34436 714	0.42721 938	0.50798 838
65	0	0.08717 554	0.17380 680	0.25935 592	0.34329 797	0.42512 769	0.50436 656
75	0	0.08716 317	0.17370 770	0.25902 064	0.34250 043	0.42356 271	0.50164 622
85	0	0.08715 659	0.17365 493	0.25884 192	0.34207 467	0.42272 556	0.50018 720

See Example 14.

Compiled from K. Pearson, Tables of the complete and incomplete elliptic integrals, Cambridge Univ. Press, Cambridge, England, 1934 (with permission). Known errors have been corrected.

ELLIPTIC INTEGRAL OF THE SECOND KIND $E(\varphi|\alpha)$ Table 17.6

$$E(\varphi|\alpha) = \int_0^\varphi (1 - \sin^2 \alpha \sin^2 \theta)^{\frac{1}{2}} d\theta$$

$\alpha \backslash \varphi$	35°		40°		45°		50°		55°		60°	
0°	0.61086	524	0.69813	170	0.78539	816	0.87266	463	0.95993	109	1.04719	755
2	0.61082	230	0.69806	905	0.78531	125	0.87254	883	0.95978	184	1.04701	051
4	0.61069	365	0.69788	136	0.78505	085	0.87220	183	0.95933	459	1.04644	996
6	0.61047	983	0.69756	935	0.78461	792	0.87162	487	0.95859	083	1.04551	764
8	0.61018	171	0.69713	427	0.78401	409	0.87081	998	0.95755	301	1.04421	646
10	0.60980	055	0.69657	784	0.78324	162	0.86979	001	0.95622	460	1.04255	047
12	0.60933	793	0.69590	226	0.78230	343	0.86853	863	0.95461	005	1.04052	491
14	0.60879	577	0.69511	023	0.78120	308	0.86707	031	0.95271	478	1.03814	615
16	0.60817	636	0.69420	492	0.77994	473	0.86539	034	0.95054	522	1.03542	177
18	0.60748	229	0.69318	999	0.77853	323	0.86350	481	0.94810	378	1.03236	049
20	0.60671	652	0.69206	954	0.77697	402	0.86142	062	0.94541	386	1.02897	221
22	0.60588	229	0.69084	814	0.77527	316	0.85914	545	0.94246	984	1.02526	804
24	0.60498	319	0.68953	083	0.77343	735	0.85668	781	0.93928	709	1.02126	023
26	0.60402	308	0.68812	308	0.77147	387	0.85405	695	0.93587	699	1.01696	224
28	0.60300	616	0.68663	077	0.76939	059	0.85126	295	0.93225	186	1.01238	873
30	0.60193	687	0.68506	023	0.76719	599	0.84831	663	0.92842	504	1.00755	556
32	0.60081	994	0.68341	817	0.76489	908	0.84522	958	0.92441	083	1.00247	977
34	0.59966	035	0.68171	170	0.76250	947	0.84201	414	0.92022	452	0.99717	966
36	0.59846	332	0.67994	830	0.76003	726	0.83868	340	0.91588	234	0.99167	469
38	0.59723	431	0.67813	578	0.75749	309	0.83525	115	0.91140	150	0.98598	560
40	0.59597	897	0.67628	229	0.75488	809	0.83173	189	0.90680	017	0.98013	430
42	0.59470	312	0.67439	630	0.75223	383	0.82814	080	0.90209	742	0.97414	397
44	0.59341	278	0.67248	651	0.74954	234	0.82449	369	0.89731	325	0.96803	899
46	0.59211	406	0.67056	191	0.74682	605	0.82080	700	0.89246	858	0.96184	497
48	0.59081	324	0.66863	167	0.74409	773	0.81709	775	0.88758	513	0.95558	873
50	0.58951	664	0.66670	515	0.74137	047	0.81338	346	0.88268	551	0.94929	830
52	0.58823	065	0.66479	183	0.73865	766	0.80968	217	0.87779	305	0.94300	285
54	0.58696	171	0.66290	130	0.73597	286	0.80601	230	0.87293	184	0.93673	272
56	0.58571	622	0.66104	317	0.73332	979	0.80239	262	0.86812	660	0.93051	931
58	0.58450	056	0.65922	707	0.73074	229	0.79884	217	0.86340	261	0.92439	505
60	0.58332	103	0.65746	255	0.72822	416	0.79538	015	0.85878	561	0.91839	329
62	0.58218	382	0.65575	905	0.72578	915	0.79202	582	0.85430	169	0.91254	821
64	0.58109	497	0.65412	585	0.72345	085	0.78879	839	0.84997	709	0.90689	460
66	0.58006	032	0.65257	197	0.72122	260	0.78571	685	0.84583	811	0.90146	778
68	0.57908	549	0.65110	612	0.71911	737	0.78279	987	0.84191	082	0.89630	323
70	0.57817	584	0.64973	667	0.71714	767	0.78006	562	0.83822	090	0.89143	642
72	0.57733	641	0.64847	154	0.71532	545	0.77753	157	0.83479	335	0.88690	237
74	0.57657	189	0.64731	812	0.71366	196	0.77521	434	0.83165	223	0.88273	530
76	0.57588	663	0.64628	328	0.71216	766	0.77312	952	0.82882	031	0.87896	810
78	0.57528	450	0.64537	322	0.71085	210	0.77129	143	0.82631	879	0.87563	185
80	0.57476	897	0.64459	347	0.70972	381	0.76971	298	0.82416	694	0.87275	520
82	0.57434	302	0.64394	879	0.70879	019	0.76840	544	0.82238	177	0.87036	381
84	0.57400	912	0.64344	316	0.70805	745	0.76737	830	0.82097	770	0.86847	970
86	0.57376	921	0.64307	973	0.70753	050	0.76663	912	0.81996	631	0.86712	068
88	0.57362	470	0.64286	075	0.70721	289	0.76619	339	0.81935	604	0.86629	990
90	0.57357	644	0.64278	761	0.70710	678	0.76604	444	0.81915	204	0.86602	540
	$\left[\begin{smallmatrix} (-5)1 \\ 5 \end{smallmatrix} \right]$		$\left[\begin{smallmatrix} (-5)2 \\ 5 \end{smallmatrix} \right]$		$\left[\begin{smallmatrix} (-5)3 \\ 5 \end{smallmatrix} \right]$		$\left[\begin{smallmatrix} (-5)4 \\ 6 \end{smallmatrix} \right]$		$\left[\begin{smallmatrix} (-5)5 \\ 6 \end{smallmatrix} \right]$		$\left[\begin{smallmatrix} (-5)7 \\ 6 \end{smallmatrix} \right]$	
5	0.61059	734	0.69774	083	0.78485	586	0.87194	199	0.95899	964	1.04603	012
15	0.60849	557	0.69467	152	0.78059	337	0.86625	642	0.95166	385	1.03682	664
25	0.60451	051	0.68883	790	0.77247	109	0.85539	342	0.93760	971	1.01914	662
35	0.59906	618	0.68083	664	0.76128	304	0.84036	234	0.91807	186	0.99445	152
45	0.59276	408	0.67152	549	0.74818	650	0.82265	424	0.89489	714	0.96495	146
55	0.58633	563	0.66196	758	0.73464	525	0.80419	500	0.87052	066	0.93361	692
65	0.58057	051	0.65333	844	0.72232	215	0.78723	820	0.84788	276	0.90415	063
75	0.57621	910	0.64678	548	0.71289	304	0.77414	195	0.83019	625	0.88079	972
85	0.57387	732	0.64324	351	0.70776	799	0.76697	232	0.82042	232	0.86773	361

Table 17.6

ELLIPTIC INTEGRAL OF THE SECOND KIND $E(\varphi|\alpha)$

$$E(\varphi|\alpha) = \int_0^\varphi (1 - \sin^2 \alpha \sin^2 \theta)^{\frac{1}{2}} d\theta$$

$\alpha \backslash \varphi$	65°	70°	75°	80°	85°	90°
0°	1.13446 401	1.22173 048	1.30899 694	1.39626 340	1.48352 986	1.57079 633
2	1.13423 517	1.22145 628	1.30867 442	1.39589 024	1.48310 448	1.57031 792
4	1.13354 929	1.22063 443	1.30770 767	1.39477 165	1.48182 929	1.56888 372
6	1.13240 837	1.21926 717	1.30609 916	1.39291 030	1.47970 717	1.56649 679
8	1.13081 573	1.21735 820	1.30385 297	1.39031 062	1.47674 288	1.56316 223
10	1.12877 602	1.21491 274	1.30097 484	1.38697 886	1.47294 312	1.55888 720
12	1.12629 522	1.21193 748	1.29747 215	1.38292 302	1.46831 652	1.55368 089
14	1.12338 066	1.20844 065	1.29335 393	1.37815 292	1.46287 363	1.54755 458
16	1.12004 099	1.20443 195	1.28863 089	1.37268 017	1.45662 693	1.54052 157
18	1.11628 624	1.19992 262	1.28331 541	1.36651 823	1.44959 085	1.53259 729
20	1.11212 778	1.19492 542	1.27742 153	1.35968 233	1.44178 179	1.52379 921
22	1.10757 834	1.18945 465	1.27096 502	1.35218 961	1.43321 813	1.51414 692
24	1.10265 204	1.18352 618	1.26396 337	1.34405 903	1.42392 023	1.50366 214
26	1.09736 439	1.17715 743	1.25643 578	1.33531 146	1.41391 049	1.49236 871
28	1.09173 228	1.17036 745	1.24840 326	1.32596 967	1.40321 335	1.48029 266
30	1.08577 404	1.16317 686	1.23988 858	1.31605 841	1.39185 532	1.46746 221
32	1.07950 942	1.15560 796	1.23091 635	1.30560 436	1.37986 503	1.45390 780
34	1.07295 961	1.14768 469	1.22151 305	1.29463 629	1.36727 328	1.43966 215
36	1.06614 728	1.13943 273	1.21170 705	1.28318 499	1.35411 306	1.42476 031
38	1.05909 660	1.13087 946	1.20152 870	1.27128 343	1.34041 965	1.40923 972
40	1.05183 322	1.12205 408	1.19101 036	1.25896 675	1.32623 066	1.39314 025
42	1.04438 435	1.11298 760	1.18018 648	1.24627 240	1.31158 614	1.37650 433
44	1.03677 875	1.10371 291	1.16909 366	1.23324 019	1.29652 865	1.35937 700
46	1.02904 677	1.09426 484	1.15777 077	1.21991 241	1.28110 340	1.34180 606
48	1.02122 034	1.08468 023	1.14625 899	1.20633 398	1.26535 837	1.32384 218
50	1.01333 305	1.07499 796	1.13460 200	1.19255 255	1.24934 449	1.30553 909
52	1.00542 010	1.06525 908	1.12284 604	1.17861 873	1.23311 580	1.28695 374
54	0.99751 835	1.05550 682	1.11104 010	1.16458 621	1.21672 971	1.26814 653
56	0.98966 632	1.04578 671	1.09923 604	1.15051 210	1.20024 724	1.24918 162
58	0.98190 414	1.03614 663	1.08748 883	1.13645 710	1.18373 339	1.23012 722
60	0.97427 354	1.02663 689	1.07585 669	1.12248 590	1.16725 747	1.21105 603
62	0.96681 780	1.01731 023	1.06440 132	1.10866 752	1.15089 364	1.19204 568
64	0.95958 158	1.00822 192	1.05318 814	1.09507 580	1.13472 145	1.17317 938
66	0.95261 084	0.99942 966	1.04228 653	1.08178 986	1.11882 658	1.15454 668
68	0.94595 256	0.99099 354	1.03176 998	1.06889 476	1.10330 172	1.13624 437
70	0.93965 447	0.98297 583	1.02171 634	1.05648 221	1.08824 773	1.11837 774
72	0.93376 462	0.97544 068	1.01220 781	1.04465 133	1.07377 505	1.10106 217
74	0.92833 088	0.96845 360	1.00333 091	1.03350 951	1.06000 556	1.08442 522
76	0.92340 024	0.96208 074	0.99517 606	1.02317 331	1.04707 504	1.06860 953
78	0.91901 802	0.95638 776	0.98783 670	1.01376 904	1.03513 640	1.05377 692
80	0.91522 691	0.95143 847	0.98140 781	1.00543 295	1.02436 393	1.04011 440
82	0.91206 588	0.94729 297	0.97598 331	0.99831 000	1.01495 896	1.02784 362
84	0.90956 905	0.94400 544	0.97165 228	0.99255 019	1.00715 650	1.01723 692
86	0.90776 445	0.94162 171	0.96849 392	0.98830 025	1.00123 026	1.00864 796
88	0.90667 305	0.94017 677	0.96657 142	0.98568 915	0.99748 392	1.00258 409
90	0.90630 779	0.93969 262	0.96592 583	0.98480 775	0.99619 470	1.00000 000
	$\begin{bmatrix} (-5)9 \\ 6 \end{bmatrix}$	$\begin{bmatrix} (-4)1 \\ 7 \end{bmatrix}$	$\begin{bmatrix} (-4)2 \\ 7 \end{bmatrix}$	$\begin{bmatrix} (-4)2 \\ 9 \end{bmatrix}$	$\begin{bmatrix} (-4)3 \\ 9 \end{bmatrix}$	$\begin{bmatrix} (-4)4 \\ 10 \end{bmatrix}$
5	1.13303 553	1.22001 878	1.30698 342	1.39393 358	1.48087 384	1.56780 907
15	1.12176 337	1.20649 962	1.29106 728	1.37550 358	1.45984 990	1.54415 050
25	1.10005 236	1.18039 569	1.26026 405	1.33976 099	1.41900 286	1.49811 493
35	1.06958 479	1.14359 813	1.21665 853	1.28896 903	1.36076 208	1.43229 097
45	1.03292 660	1.09900 829	1.16345 846	1.22661 050	1.28885 906	1.35064 388
55	0.99358 365	1.05063 981	1.10513 448	1.15755 065	1.20849 656	1.25867 963
65	0.95606 011	1.00378 508	1.04769 389	1.08838 943	1.12673 373	1.16382 796
75	0.92579 978	0.96518 626	0.99915 744	1.02823 305	1.05342 632	1.07640 511
85	0.90857 873	0.94269 813	0.96992 212	0.99022 779	1.00394 027	1.01266 351